

# Challenge

Spare Time Teaching

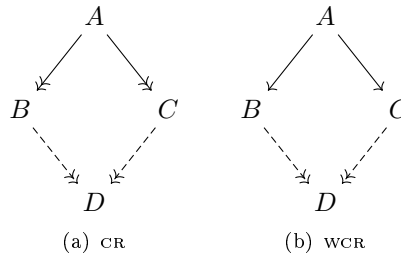
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## Problem

We say that a system is *Church-Rosser* (CR, also known as *confluent*) if: for all paths  $A \rightarrow^* B$  and  $A \rightarrow^* C$ , there exists paths  $B \rightarrow^* D$  and  $C \rightarrow^* D$ .

We say that a system is *weak Church-Rosser* (WCR) if: for all steps  $A \rightarrow B$  and  $A \rightarrow C$ , there exists paths  $B \rightarrow^* D$  and  $C \rightarrow^* D$ .

Visualized:



It is clear that  $CR \Rightarrow WCR$ . But what about the other way around?

Prove  $WCR \Rightarrow CR$  or make a counter example (a system that is WCR but not CR).

## Example

$\rightarrow^\beta$  (beta-reduction) in Lambda Calculus is CR (therefore also WCR).

Here is another system that is also CR:

